Motion Models (cont)

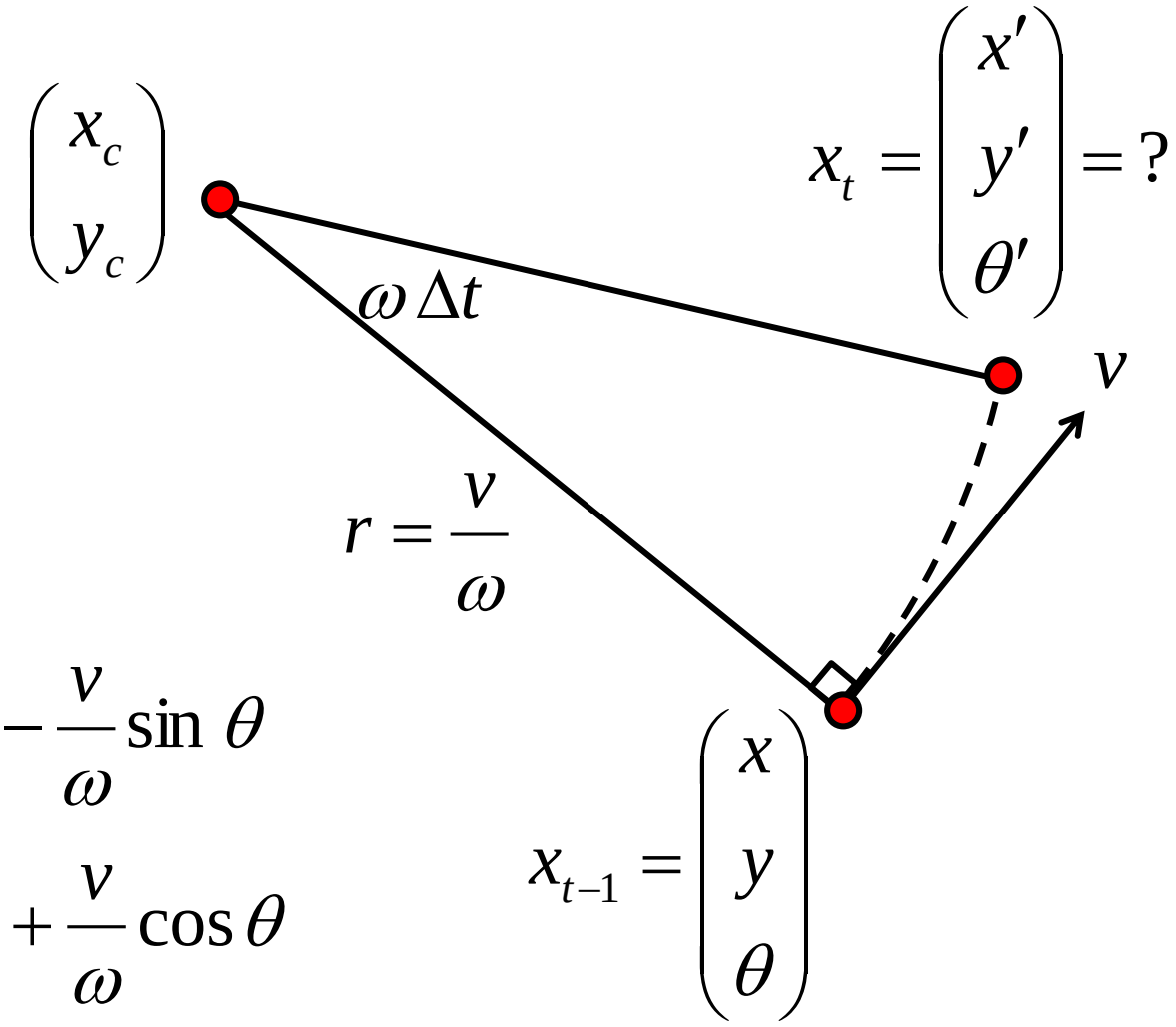
Sampling from the Velocity Motion Model

- suppose that a robot has a map of its environment and it needs to find its pose in the environment
  - this is the robot localization problem
  - several variants of the problem
    - the robot knows where it is initially
    - the robot does not know where it is initially
    - kidnapped robot: at any time, the robot can be teleported to another location in the environment
- a popular solution to the localization problem is the particle filter
  - uses simulation to sample the state density $p(x_t \mid u_t, x_{t-1})$

Sampling from the Velocity Motion Model

- sampling the conditional density is easier than computing the density because we only require the forward kinematics model
  - given the control u_t and the previous pose x_{t-1} find the new pose x_t

Sampling from the Velocity Motion Model



$$x_c = x - \frac{v}{\omega}\sin\theta$$

$$y_c = y + \frac{v}{\omega}\cos\theta$$

Eqs 5.7, 5.8

Sampling from the Velocity Motion Model

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x_c + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \\ y_c - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \\ \theta + \omega\Delta t \end{pmatrix}$$

$$= \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega}\sin\theta + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \\ \frac{v}{\omega}\cos\theta - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \\ \omega\Delta t \end{pmatrix} \quad \text{Eqs 5.9}$$

*we already derived this for the differential drive!

Sampling from the Velocity Motion Model

- as with the original motion model, we will assume that given noisy velocities the robot can also make a small rotation in place to determine the final orientation of the robot

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{\omega}}\sin\theta + \frac{\hat{v}}{\hat{\omega}}\sin(\theta + \hat{\omega}\Delta t) \\ \frac{\hat{v}}{\hat{\omega}}\cos\theta - \frac{\hat{v}}{\hat{\omega}}\cos(\theta + \hat{\omega}\Delta t) \\ \hat{\omega}\Delta t + \hat{\gamma}\Delta t \end{pmatrix}$$

Sampling from the Velocity Motion Model

1: **Algorithm sample_motion_model_velocity**(u_t, x_{t-1}):

2: $\hat{v} = v + \textbf{sample}(\alpha_1 \, v^2 + \alpha_2 \, \omega^2)$

3: $\hat{\omega} = \omega + \textbf{sample}(\alpha_3 \, v^2 + \alpha_4 \, \omega^2)$

4: $\hat{\gamma} = \textbf{sample}(\alpha_5 \, v^2 + \alpha_6 \, \omega^2)$

5: $x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}} \, sin(\theta + \hat{\omega}\Delta t)$

6: $y' = y + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}} \, cos(\theta + \hat{\omega}\Delta t)$

7: $\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$

8: *return* $x_t = (x', y', \theta')^T$

Sampling from the Velocity Motion Model

- the function $\text{sample}(b^2)$ generates a random sample from a zero-mean distribution with variance b^2
- Matlab is able to generate random numbers from many different distributions
  - help randn
  - help stats

How to Sample from Normal or Triangular Distributions?

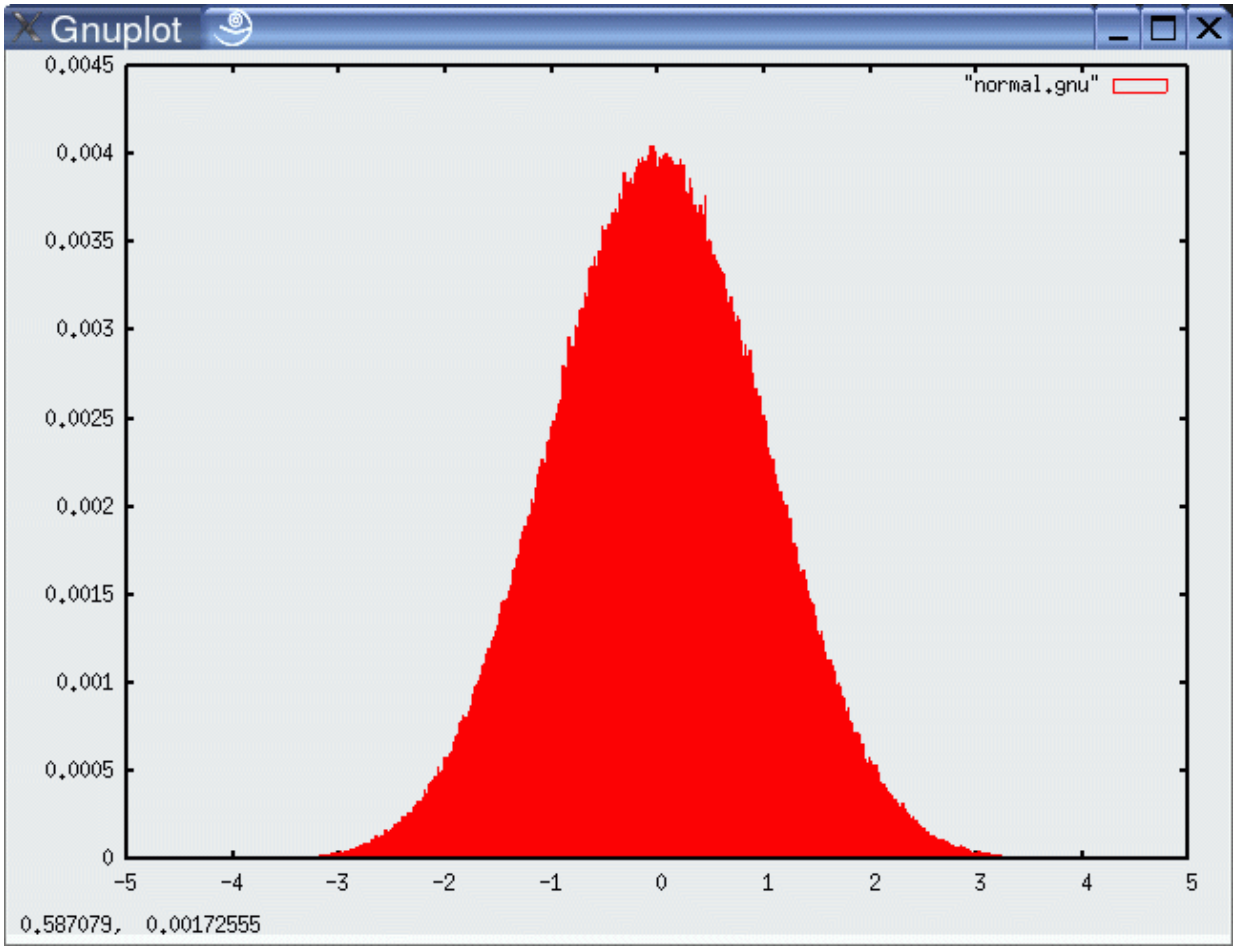
- Sampling from a normal distribution

1. Algorithm **sample_normal_distribution**(b):
2. return $\frac{1}{2}\sum_{i=1}^{12} rand(-b, b)$

- Sampling from a triangular distribution

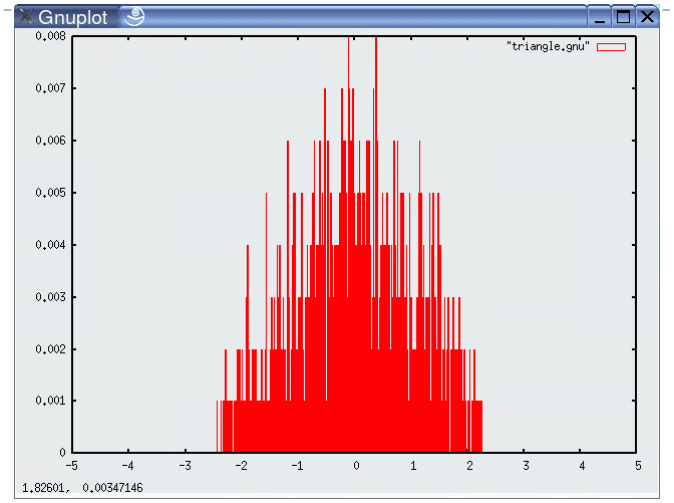
1. Algorithm **sample_triangular_distribution**(b):
2. return $\frac{\sqrt{6}}{2}\left[\mathbf{rand}(-b, b) + \mathbf{rand}(-b, b)\right]$

Normally Distributed Samples

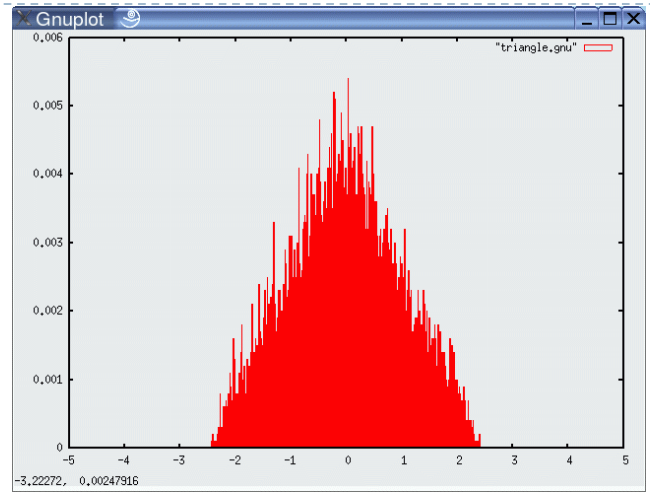


10^6 samples

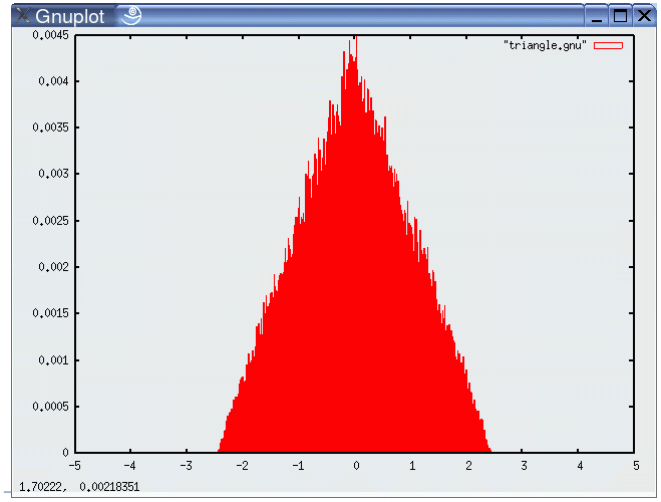
For Triangular Distribution



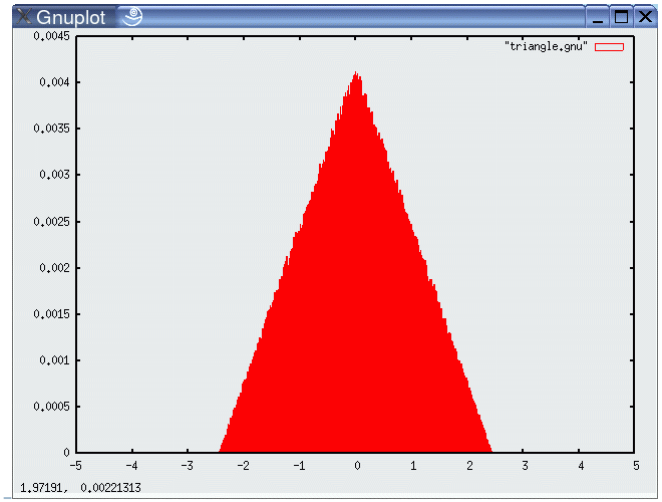
10^3 samples



10^4 samples



10^5 samples



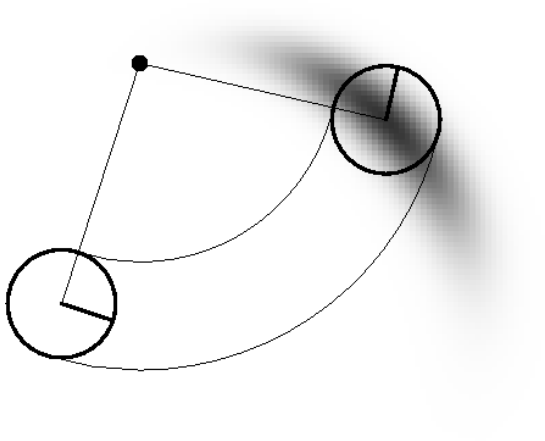
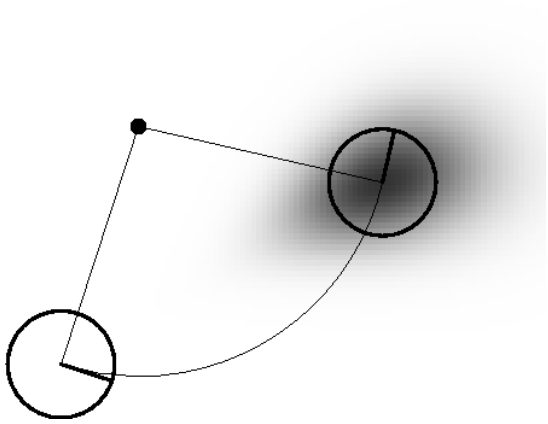
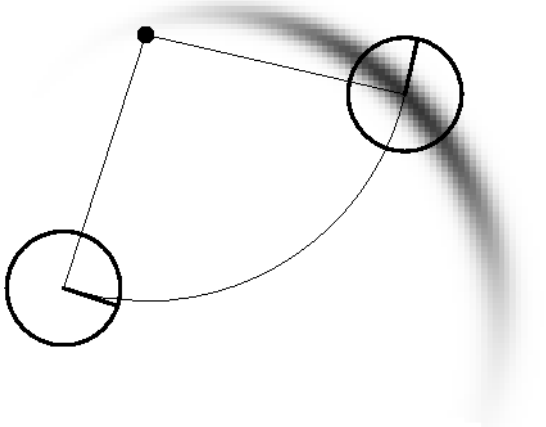
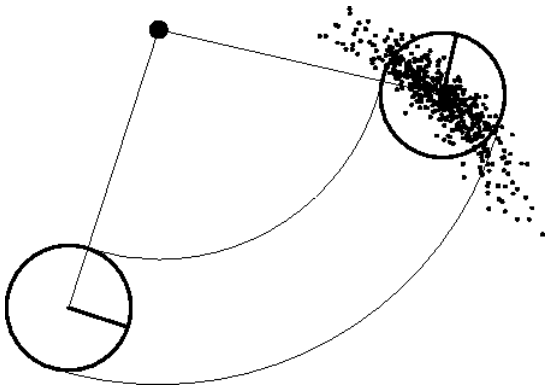
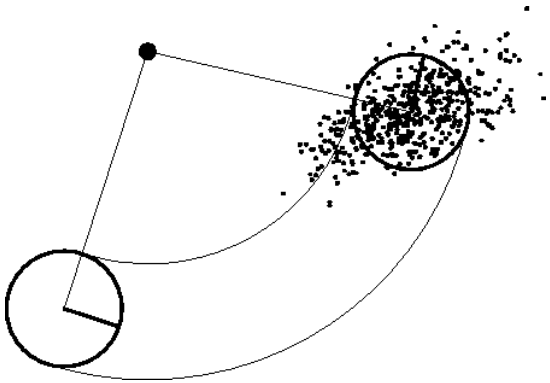
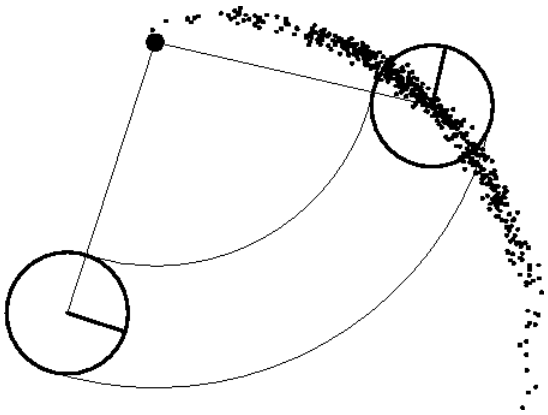
10^6 samples

Rejection Sampling

- Sampling from arbitrary distributions

1. Algorithm **sample_distribution**(*f*,*b*):
2. repeat
3. $x = \mathbf{rand}(-b, b)$
4. $y = \mathbf{rand}(0, \max\{f(x) \mid x \in (-b, b)\})$
5. until ($y \leq f(x)$
6. return x

Examples

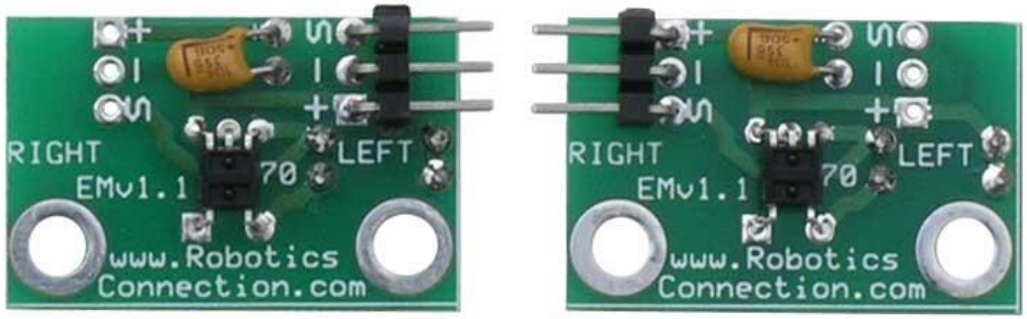







Odometry Motion Model

- many robots make use of odometry rather than velocity
- odometry uses a sensor or sensors to measure motion to estimate changes in position over time
- typically more accurate than velocity motion model, but measurements are available only after the motion has been completed
- technically a measurement rather than a control
  - but usually treated as control to simplify the modeling
- odometry allows a robot to estimate its pose
  - but no fixed mapping from odometer coordinates and world coordinates
- in wheeled robots the sensor is often a rotary encoder

Example Wheel Encoders

These modules require +5V and GND to power them, and provide a 0 to 5V output. They provide +5V output when they "see" white, and a 0V output when they "see" black.



These disks are manufactured out of high quality laminated color plastic to offer a very crisp black to white transition. This enables a wheel encoder sensor to easily see the transitions.

Source: http://www.active-robots.com/

Odometry Model

bar indicates odometer coordinates

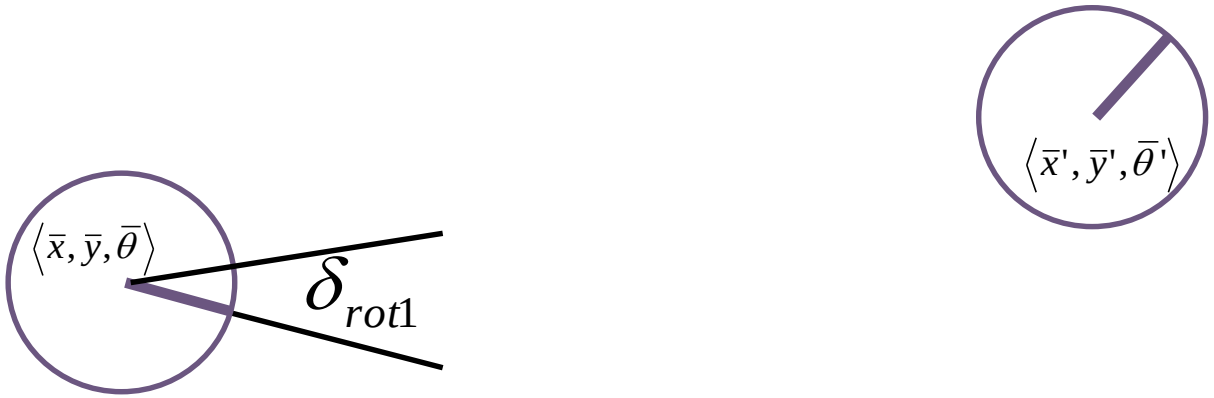
- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.

Odometry Model

bar indicates odometer coordinates

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.

Step 1: rotate in place by δ_{rot1}



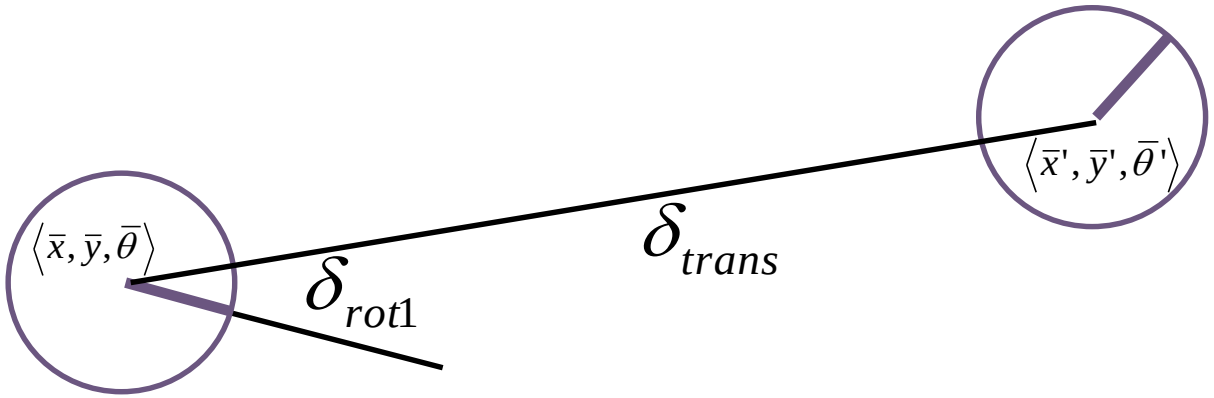
Odometry Model

bar indicates odometer coordinates

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.

Step 1: rotate in place by δ_{rot1}

Step 2: move to $\langle \bar{x}', \bar{y}' \rangle$



Odometry Model

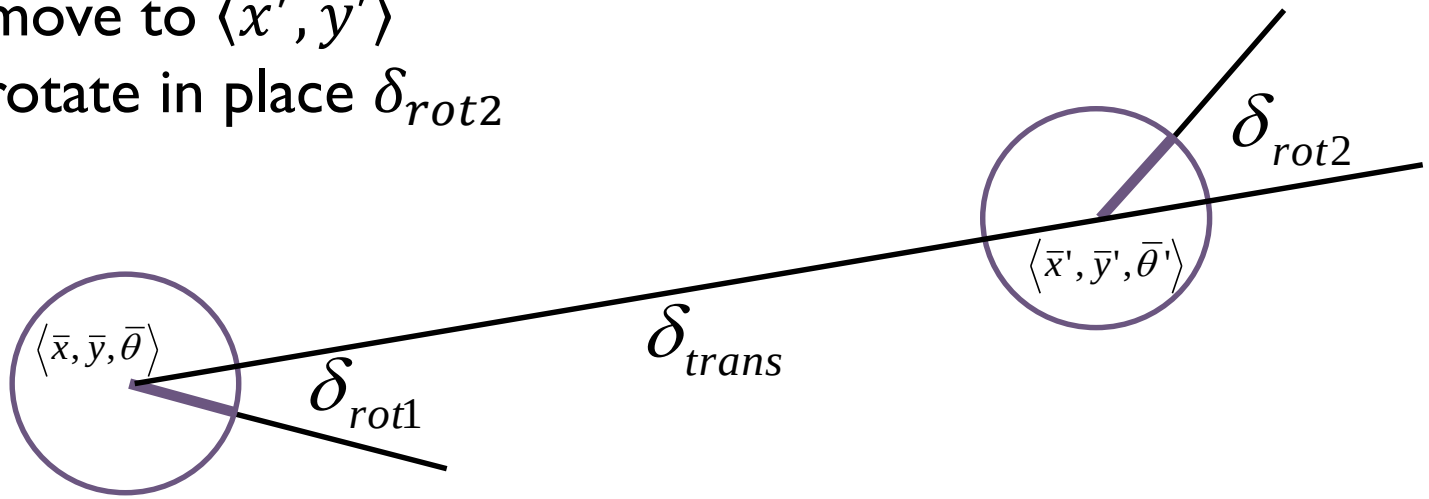
bar indicates odometer coordinates

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.

Step 1: rotate in place by δ_{rot1}

Step 2: move to $\langle \bar{x}', \bar{y}' \rangle$

Step 3: rotate in place δ_{rot2}



Odometry Model

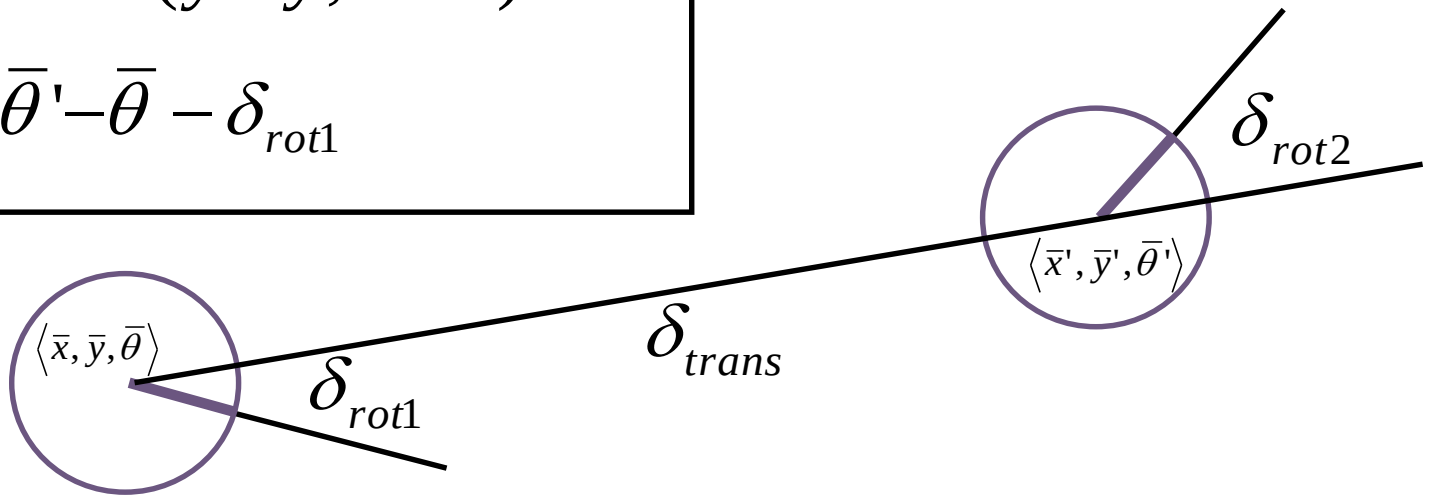
bar indicates odometer coordinates

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$

$$\delta_{trans} = \sqrt{(\bar{x}'-\bar{x})^2 + (\bar{y}'-\bar{y})^2}$$

$$\delta_{rot1} = \text{atan2}(\bar{y}'-\bar{y}, \bar{x}'-\bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}'-\bar{\theta} - \delta_{rot1}$$



Noise Model for Odometry

- The measured motion is given by the true motion corrupted with noise.

$$\hat{\delta}_{rot1} = \delta_{rot1} - \varepsilon_{\alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2}$$

$$\hat{\delta}_{trans} = \delta_{trans} - \varepsilon_{\alpha_3 \delta_{trans}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{rot2}^2)}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} - \varepsilon_{\alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2}$$

Sample Odometry Motion Model

1. Algorithm **sample_motion_model**(u, x):

$$u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$$

1. $\hat{\delta}_{rot1} = \delta_{rot1} + \text{sample}(\alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2)$
2. $\hat{\delta}_{trans} = \delta_{trans} + \text{sample}(\alpha_3 \delta_{trans}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{trans}^2))$
3. $\hat{\delta}_{rot2} = \delta_{rot2} + \text{sample}(\alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2)$
4. $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5. $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6. $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

7. Return $\langle x', y', \theta' \rangle$

sample_normal_distribution

Sampling from Our Motion Model